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Free Convective MHD Jeffrey Fluid Flow between Two Coaxial Inclined Permeable Cylinders

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Abstract

MHD free convective flow of a Jeffery fluid between two coaxial inclined cylinders has been investigated by using the quadratic density temperature (Q.D.T) relationship. The inner and outer cylinders are permeable. Applying perturbation method the velocity and the temperature fields are obtained. The effects of heat source parameter, permeability parameter, Jeffrey parameter and Hartmann number on the velocity and temperature distributions are discussed in detail. The magnitude of velocity increases with the increasing permeability parameter σ whereas the magnetic field reduces the velocity in the annulus of two permeable cylinders.

Keywords: Inclined channel, Coaxial Cylinders, Permeability, Jeffrey fluid.

Introduction

The flow and heat transfer of electrically conducting fluids in channels and circular cylinders under the effect of a radial magnetic field occurs in magnetohydrodynamic (MHD) generators, pumps, accelerators, and flow meters and have applications in nuclear reactors, filtration, geothermal systems, and others. Free convection is a very important mechanism that is operative in a variety of environments from cooling electronic circuit boards in computers to causing large scale circulation in the atmosphere as well as in lakes and oceans that influences the weather. It is caused by the action of density gradients in conjunction with gravitational field. Free convection flows in tubes and channels bounded by permeable walls have been the subject of research for many years. Most of the works reported are restricted to the case of impermeable wall. Soundalgekar [1] have studied free convection effects on steady MHD flow past a vertical porous plate. Vajravelu and Sastri [2] and Das and Ahamed [3] considered the problem of free convective heat transfer in a viscous incompressible fluid confined between a vertical wavy wall and a flat wall. Laminar free convection flow with and without heat sources through coaxial circular pipes has been studied by Gupta et al. [4]. However in all these investigations linear density temperature variation is assumed to express the body force term as buoyancy term. Goren [5] has suggested a quadratic density temperature distribution, given by

$$\Delta\rho = -\rho\beta\left(T - T_s\right)^2 \qquad (1)$$

where ρ is the density, β is the constant and T_s is the temperature in hydrostatic condition.

Such nonlinear relationship between density and temperature may be useful to explain the anomalous behavior of water at 4^0c. Balakrishan et al. [6] used this density temperature relation to investigate the MHD flow between coaxial cylinders.

Chamkha [7] studied the problem of unsteady, two-dimensional, laminar, boundary-layer flow of a viscous, incompressible, electrically conducting and heat-absorbing fluid along a semi- infinite vertical permeable moving plate in the presence of a uniform transverse magnetic field and thermal and concentration buoyancy effects. Makinde [8] examined the transient free convection interaction with thermal radiation of an absorbing-emitting fluid along moving vertical permeable plate. Tak and Kumar [9] presented MHD free convection flow with viscous dissipation in a vertical wavy channel.

Aydin and Avci [10] investigated analytically to predict laminar heat convection in a Couette–Poiseuille flow between two parallel plates with a simultaneous pressure gradient and an axial movement of the upper plate. The effect of the modified Brinkman number on the temperature distribution and the Nusselt number has been discussed for different values of the relative velocity of the upper plate. The problem of natural

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convection from a vertical wavy plate embedded in porous media for power law fluids in presence of magnetic field is studied by Mahdy et al. [11] and mixed convection heat and mass transfer on a vertical wavy plate embedded in a saturated porous media is analysed by Mahdy [12]. Krishna Gopal Singha [13] investigated analytical solution to the problem of MHD free convective flow of an electrically conducting fluid between two heated parallel plates in the presence of an induced magnetic field. Sreenadh et al. [14] obtained a solution for the MHD free convective flow of a Jeffrey fluid between coaxial cylinders using QDT relation. The problem is solved using a perturbation technique. The effects of various physical parameters on the flow characteristics are discussed.

In this paper, free convective flow of a Jeffrey fluid between two inclined coaxial circular cylinders is investigated in the presence of a radial magnetic field. The inner outer cylinders are is permeable. The velocity, the temperature and the Nusselt number are determined and results are discussed through graphs.

Formulation of the Problem

Let us consider the fully developed steady laminar free convective flow of a Jeffrey fluid between coaxial circular cylinders with permeable walls in the presence of a radial magnetic field (see Fig.1). The radii of inner and outer cylinders are a and b ($< a$) respectively. The flow in the porous medium is given by Darcy's law whereas the flow in the annulus is described by Jeffrey model. We consider only the axial flow and since the pipes are long enough, the flow depends only on r, so the velocity vector is of the form (0,0,w(r)).

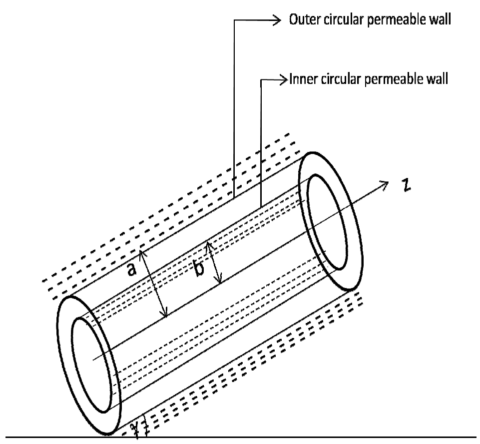

Fig.1 Physical Model

The equations of motion and energy are

$$0=-\frac{\partial p}{\partial z}+\frac{\mu}{1+\lambda_1}\left[\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right]-\frac{\sigma_0\mu_e^2H_0^2a^2}{r^2}w+\rho g_1\sin\gamma$$

(2)

$$0=K_1\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)+\mu\left(\frac{\partial w}{\partial r}\right)^2+Q$$

(3)

Where $g_1=-g$ and Q is a constant, denotes the heat added due to heat sources, $g_1\sin\gamma$ the generating body force, p the pressure, μ the coefficient of viscosity, λ_1 is the Jeffrey parameter , σ_0 the electrical conductivity, μ_e the magnetic permeability, H_0 the magnetic field, K_1 the coefficient of thermal conductivity, δ is the angle of inclination.

The boundary conditions are:

at r=a, $\frac{\partial w}{\partial r}=-\frac{\alpha}{\sqrt{k_0}}w$ (Saffman(1971)slip condition), $T=T_{w_1}$ (4)

at r=b $\frac{\partial w}{\partial r}=\frac{\alpha}{\sqrt{k_0}}w$ $T=T_{w2}$

(5)

where α is the slip parameter, k_0 is the permeability

Following Ostrach (1952), the body force in (2) can be expressed as a buoyancy term. In the hydrostatic condition equation (2) gives

$$\rho_s g_1\sin\gamma-\frac{\partial p_s}{\partial z}=0$$

(6)

and hence

$$\rho g_1\sin\gamma-\frac{\partial p}{\partial z}=\rho g_1\sin\gamma-\rho_s g_1\sin\gamma+\rho_s g_1\sin\gamma-\frac{\partial p}{\partial z}$$

$$=\left(\rho-\rho_s\right)g_1\sin\gamma-\frac{\partial}{\partial z}\left(p-p_s\right)$$

$$=-\rho\beta g_1\theta^2\sin\gamma-\frac{\partial p_0}{\partial z}$$

(7)

where $p_0=p-p_s$ and $\theta=T-T_s$

We introduce the following flow quantities in order to make the basic equations and boundary conditions dimensionless

$$\eta=\frac{r}{a},\theta^*=\frac{K\theta}{\theta_s},w^*=\frac{Kw}{W},W=\frac{g_1\beta^2a^2\theta_s^2}{v},K=\frac{g_1^2\beta^2\rho^2a^4\theta_s^3}{\mu K_1},$$

(free convection parameter) $\beta=\dfrac{a^2Q}{K_1\theta_s}$ (heat source parameter)

(8) where θ_s is the average wall temperature .

In view of (8), equations (2) – (4) reduce to the following non-dimensional form of equations. Neglecting asterisks(*),we get

$$\frac{d^2w}{d\eta^2}+\frac{1}{\eta}\frac{dw}{d\eta}-\left(\frac{1+\lambda_1}{\eta^2}\right)M^2w-\left(\frac{1+\lambda_1}{K}\right)\theta^2\sin\gamma=0 \quad (9)$$

$$\frac{d^2\theta}{d\eta^2}+\frac{1}{\eta}\frac{d\theta}{d\eta}+\left(\frac{dw}{d\eta}\right)^2+\beta K=0 \quad (10)$$

Here K can also be expressed as $K=G_rP_r\beta g_1a\theta_s/c_p$ in which $G_r=\beta g_1a^3\theta_s^2/v^2$ is the Grashoff number and $P_r=\dfrac{\mu c_p}{k_1}$ is the Prandtl number, $M=\mu_eH_0a\sqrt{\dfrac{\sigma_0}{\mu}}$ is the Hartmann number.

The corresponding boundary conditions are:

$$at\quad \eta=1,\quad w=\frac{-1}{\alpha\sigma_1}\frac{dw}{d\eta},\quad \theta=KN_1 \quad (11)$$

$$at\quad \eta=b/a,\quad w=\frac{1}{\alpha\sigma_2}\frac{dw}{d\eta},\quad \theta=KN_2 \quad (12)$$

where

$$N_1=\frac{Tw_1-T_z}{\theta_s}\quad and\quad N_2=\frac{Tw_2-T_s}{\theta_s}$$

Solution of the Problem

For the solution of the equations (9) and (10) we assume

$$w=Kw_0+K^2w_1+K^2w_2+\ldots\ldots\ \ldots\ldots \quad (13)$$

and

$$\theta=K\theta_0+K^2\theta_1+K^2\theta_2+\ldots\ldots\ \ldots\ldots \quad (14)$$

Substituting (13) and (14) into (9) and (10) and equating the coefficients of like powers of K, we get

$$w_0''+\frac{1}{\eta}w_0'-\left(\frac{1+\lambda_1}{\eta^2}\right)M^2w_0-(1+\lambda_1)\sin\gamma\theta_0^2=0 \quad (15)$$

$$w_1''+\frac{1}{\eta}w_1'-\left(\frac{1+\lambda_1}{\eta^2}\right)M^2w_1-2(1+\lambda_1)\sin\gamma\theta_0\theta_1=0 \quad (16)$$

…. ….. …..

$$\theta_0''+(1/\eta)\theta'_0+\beta=0 \quad (17)$$

$$\theta_1''+(1/\eta)\theta'_1+w'^2_0=0 \quad (18)$$

….. ….. …...

The boundary conditions become

$$at\quad \eta=1,\quad w_0=\frac{-1}{\alpha\sigma_1}\frac{dw^*}{d\eta^*},\quad \theta_0=N_1,\quad \theta_1=0 \quad (19)$$

and

$$at\quad \eta=b/a,\quad w_0=\frac{1}{\alpha\sigma_2}\frac{dw^*}{d\eta^*},\quad \theta_0=N_2,\quad \theta_1=0 \quad (20)$$

The values of θ_0 and w_0 are obtained as

$$\theta_0=A_0+B_0\log\eta-(\beta/4)\eta^2 \quad (21)$$

and

$$w_0=C_0\eta^{M\sqrt{1+\lambda_1}}+D_0\eta^{-M\sqrt{1+\lambda_1}}+E_0\eta^2-F_0\eta^4+G_0\eta^6+[H_0-I_0\eta^2+J_0\log\eta]\eta^2\log\eta \quad (22)$$

where

$$A_0=N_1+\frac{\beta}{4},\qquad B_0=\frac{N_2-A_0+\beta\left(b^2/4a^2\right)}{\log b/a}$$

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$$E_0=\frac{(1+\lambda_1)}{4-(1+\lambda_1)M^2}\left[A_0^2-\frac{8A_0B_0}{4-(1+\lambda_1)M^2}+\frac{2B_0^2\left(12+(1+\lambda_1)M^2\right)}{4-(1+\lambda_1)M^2}\right]\sin\gamma$$

$$F_0=\frac{(1+\lambda_1)\beta}{16-(1+\lambda_1)M^2}\left[\frac{A_0}{2}-\frac{4B_0}{16-(1+\lambda_1)M^2}\right]\sin\gamma$$

$$G_0=\frac{(1+\lambda_1)\beta^2}{16\left(36-(1+\lambda_1)M^2\right)}\sin\gamma$$
, ,

$$H_0=\frac{(1+\lambda_1)}{4-(1+\lambda_1)M^2}\left[2A_0B_0-\frac{8B_0^2}{4-(1+\lambda_1)M^2}\right]\sin\gamma$$

$$I_0=\frac{(1+\lambda_1)B_0\beta}{2\left(16-(1+\lambda_1)M^2\right)}\sin\gamma$$
, ,

$$J_0=\frac{(1+\lambda_1)B_0^2}{4-(1+\lambda_1)M^2}\sin\gamma$$
,

$$a_1=M\sqrt{1+\lambda_1}+\alpha\sigma_1,\quad a_2=M\sqrt{1+\lambda_1}-\alpha\sigma_1$$

$$k_1=$$

$$E_0(2+\alpha\sigma_1)-F_0(4+\alpha\sigma_1)+G_0(6+\alpha\sigma_1)+H_0-I_0$$

$$a_3=M\sqrt{1+\lambda_1}\left(\frac{b}{a}\right)^{M\sqrt{1+\lambda_1}-1}-\alpha\sigma_2\left(\frac{b}{a}\right)^{M\sqrt{1+\lambda_1}},$$

$$a_4=M\sqrt{1+\lambda_1}\left(\frac{b}{a}\right)^{-M\sqrt{1+\lambda_1}-1}+\alpha\sigma_2\left(\frac{b}{a}\right)^{-M\sqrt{1+\lambda_1}}$$

$$K_2=\left[2E_0\left(\frac{b}{a}\right)-\alpha\sigma_2E_0\left(\frac{b}{a}\right)^2-4F_0\left(\frac{b}{a}\right)^3+\alpha\sigma_2\left(\frac{b}{a}\right)^4+6G_0\left(\frac{b}{a}\right)^5-\alpha\sigma_2\left(\frac{b}{a}\right)^6+\left(H_0-I_0\left(\frac{b}{a}\right)^2+J_0\log\frac{b}{a}\right)\left(\frac{b}{a}\right)^2\log\frac{b}{a}\right]$$

$$+\left(\frac{b}{a}\right)+2\left(\frac{b}{a}\right)\log\left(\frac{b}{a}\right)+\left[-2\left(\frac{b}{a}\right)I_0+J_0\left(\frac{b}{a}\right)\right]\left(\frac{b}{a}\right)^2\log\left(\frac{b}{a}\right)$$

$$C_0=\frac{K_2a_2-k_1a_4}{a_1a_4-a_2a_3},\qquad D_0=\frac{k_2a_1-k_1a_3}{a_1a_4-a_2a_3}$$

Similar polynomials in η can be obtained for θ_1 and w_1

The heat transfer through the pipe walls to the flow per unit area of the pipe surfaces are given by

$$q=\frac{K_1\theta_s}{aK}\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=(b/a),1} \quad (23)$$

The Nusselt number on the walls are:

$$Nu_1=\frac{aq}{K_1\theta_s}=\frac{1}{K}\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=1} \quad (24)$$

and $Nu_2=\frac{aq}{K_1\theta_s}=\frac{1}{K}\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=b/a}$

Discussion of the Results

The numerical values of velocity and temperature functions have been computed for $N_1=2, N_2=4$ and $\frac{b}{a}=0.2$. We observe from Fig.2 that the magnitude of velocity increases with the increase in the Hartmann number M. We observe the same phenomenon is observed from Fig.3 but the magnitude of velocity becomes higher for the increase in the heat source parameter.

From Fig.4, we conclude that the effect of Jeffrey parameter is more nearer to the inner wall of the cylinder. The magnitude of velocity decreases with the increase in the Jeffrey parameter λ_1. It is also noted from Fig.5 that for a given value of λ_1, the magnitude of velocity is increasing with the increment in the heat source parameter.

We observe from Fig.6 that the magnitude of velocity increases with the increasing permeability parameter σ. From Fig.7, we find that positive sign in the heat source parameter β gives rise to more velocity when compared with negative sign of β.

We observe from Figs.8 and 9 that the magnitude of velocity decreases with the increasing inclination γ. we also note that positive sign in the heat source parameter β gives rise to more velocity when compared with negative sign of β.

The temperature is numerically evaluated using equation (22) for different values of heat source parameter β and is shown in Fig.10. It seen that the temperature increases with the increasing values of β (when β> 0) and decreases with the decreasing values of β (when β< 0). We note that the there is no effect of the Jeffrey parameter on the temperature θ_0. This is because, the temperature θ_0 is independent of λ_1. The effect of non-Newtonian behavior of the Jeffrey parameter λ_1 on the temperature may be seen in the first order solution θ_1.

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We observe from Fig.11 that for given free convection parameter K, Nusselt number decreases with increasing heat source parameter β at the outer wall when the temperature currents flow from inner wall to outer wall. In view of Fig.12. we infer that the behavior of Nu_2 is otherwise owing to an increase in β at the inner wall .

The solutions for higher order can be obtained using numerical techniques like finite difference method, as exact solutions are complex in such cases.

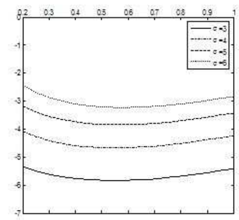

Fig.6 Velocity distribution for various values of σ for fixed $N_1 = 2$, $N_2 = 4$, $\frac{b}{a} = 0.20$, $\alpha = 0.1$, $\beta = 5$, $\lambda_1 = 0.1$, $\sigma = 5$, $M = 0.5$, $\gamma = \frac{\pi}{4}$.

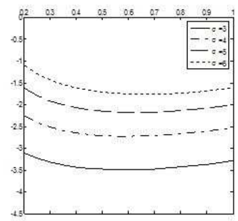

Fig.7 Velocity distribution for various values of σ for fixed $N_1 = 2$, $N_2 = 4$, $\frac{b}{a} = 0.20$, $\alpha = 0.1$, $\beta = -5$, $\lambda_1 = 0.1$, $\sigma = 5$, $M = 0.5$, $\gamma = \frac{\pi}{4}$.

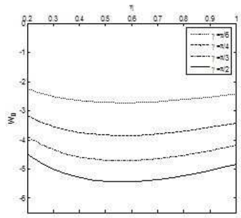

Fig.8 Velocity distribution for various values of σ for fixed $N_1 = 2$, $N_2 = 4$, $\frac{b}{a} = 0.20$, $\alpha = 0.1$, $\beta = 5$, $\lambda_1 = 0.1$, $M = 0.3$.

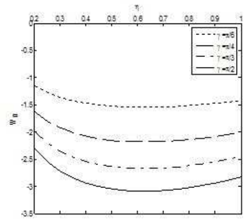

Fig.9 Velocity distribution for various values of σ for fixed $N_1 = 2$, $N_2 = 4$, $\frac{b}{a} = 0.20$, $\alpha = 0.1$, $\beta = -5$, $\lambda_1 = 0.1$, $M = 0.3$.

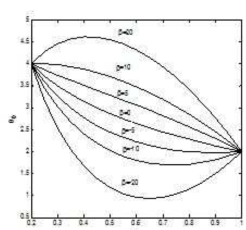

Fig.10 Temperature distribution for various values of β for fixed $N_1 = 2$, $N_2 = 4$, $\frac{b}{a} = 0.20$.

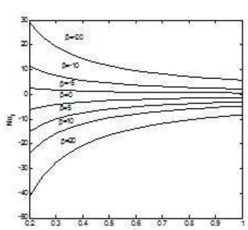

Fig.11 Nusselt number at the outer wall for various values of β for fixed N_1=2, N_2=4, $\frac{b}{a}$=0.2.

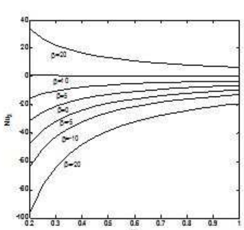

Fig.12 Nusselt number at the inner wall for various values of β for fixed N_1=2, N_2=4, $\frac{b}{a}$=0.2.

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